C^1 finite element method for streamfunction formulation of MHD equations

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This work deals with the numerical approximation of the magnetohydrodynamic (MHD) equations describing an incompressible plasma in a tokamaks (a toroidal device in which plasma is confined thanks to a very strong applied magnetic field). We assume that the flows are independent of the toroidal coordinate and the resulting model is thus 2-D. We have implemented a C^1 finite element method on general triangles : the so-called Clough Tocher finite element. The C^1 elements allow the treatment of fourth order operators that appear in our model by using a streamfunction formulation of the MHD equations. These triangular elements are more flexible for representing complex geometry such as tokamaks, and allow the use of mesh adaptation procedure. The use of stream functions provides a natural way to achieve the divergence-free constraints.

The developed method is applied to simulate MHD instabilities and plasma equilibrium in tokamaks in axisymmetric configuration. The obtained results agree with other studies and theoretical scaling results.

Key Words : reduced MHD, Streamfunction formulation, C^1 finite element, MHD equilibrium